Brief review on AdS_5/CFT_4 correspondence

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Objectives

The conjecture of this correspondence involves different observations of N D_3-branes, leading to two interpretations of the same physics. The argument would be present in following order:

- Simple Summary of Conditions and Physical Spaces
- First Perspective on Physics:
- Second Perspective on Physics:
- Deriving the AdS/CFT Correspondence from Equivalence and remarks on Interpretation and Limits

Introduction

When we see N D_3 Branes coincide with each other in 10D spacetime, we have two ways to describe this system. 1 is called open string point of view which is field side. 2 is a description in closed string which is the gravity side. We only consider energy scale $E \ll \frac{1}{\alpha'}$ and $\alpha' \to 0$ in our discussion.

Open String View

The D-brane is dynamical, and quantum fluctuations on it excite open strings. Considering the interaction between the brane and the closed strings of the background spacetime, the total action for the system should be written as

$$S = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{int}}. \quad (1)$$

- S_{bulk}:

The bulk part describes the dynamics of closed strings in a spacetime without branes. And the low energy effective action is the same SUGRA(Supergravity) action.

Since we consider an energy range where only massless fields are excited, we obtain the low-energy coupling of strings with the back ground fields. To preserve Weyl symmetry, we can derive three β functions from the energy-momentum tensor, which can serve as the equations of motion for this action.

Thus, the dynamics of closed strings in S_{bulk} is described by the low-energy effective action for closed strings.

$$S_{\text{bulk}} = \frac{1}{2\kappa_0^2}\int d^{26}X\sqrt{-G}\,e^{-2\Phi} \left(\mathcal{R} - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\,\partial^\mu\Phi\right) \quad (2)$$

- $S_{\text{brane}} + S_{\text{int}}$:

The DBI (Dirac-Born-Infeld) action describes the action of a single brane itself, along with its interaction with the background spacetime, using a low-energy effective form:

$$S_{\text{DBI}} = -T_p\int d^{p+1}\xi\; e^{-\tilde{\Phi}} \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})} \quad (3)$$

Expanding with $g \to \eta + \kappa h$, focusing on the metric perturbation term and when $\kappa \sim g_s\alpha'^2 \to 0$, we obtain free gravity theory, and any couplings involving h terms are suppressed by κ, so gravity **decouples** from the background fields in the system.

$$S_{\text{bulk}} \sim \frac{1}{2\kappa^2}\int\sqrt{g}R \sim \int(\partial h)^2 + \kappa(\partial h)^2 h + \cdots \quad (4)$$

The excitations of open strings on the D-brane at this energy scale correspond to massless gauge fields A_a. Expanding the DBI action gives the following form, where for $p = 3$ we can eliminate all α' suppression factors and obtain a free gauge field,where $T_p = (2\pi)^{-p}\alpha'^{-\frac{p+1}{2}}$

$$S_{\text{brane}} = -(2\pi\alpha')^2\frac{T_P}{4g_s}\int \mathrm{d}^{p+1}\xi \left(F_{ab}^{(c)}F_{(c)}^{ab} + \dots\right) \quad (5)$$

With N overlapping D_3-branes, the position of open string endpoints on different branes is described by Chan Paton factors, which act as labels for the endpoints of open strings. This leads to a reshuffling of brane indices corresponding to global symmetry, which is associated with local symmetry in spacetime. By substituting the indices into the fields, replacing ∂_μ with D_μ, and introducing a potential term, we obtain the Yang-Mills action part.

$$S = -\frac{1}{g_{YM}^2}\int d^{p+1}x \; \mathrm{Tr}\left(\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} - \frac{1}{2}D^\alpha\Phi^a D_\alpha\Phi^a + \left[\Phi^a,\Phi^b\right]^2 + \cdots\right) \quad (6)$$

The interaction term corresponds to the coupling between the gauge field and the dilaton, Kalb-Ramond field, or the metric field. Through the expansion $e^{-\Phi} \to g_s \times e^{-\tilde{\Phi}}$ and regularization, we need to rescale Φ by a κ factor. In the limit $\alpha' \to 0$, the interaction terms vanish, meaning that the brane **decouples** from the back ground spacetime.

Closed String View

$\alpha' = l_s^2 \to 0$ indicates that the string scale is much smaller than the spacetime curvature, allowing us to treat the N D_3-branes as a source in spacetime. In the limit $g_s \to 0$, we neglect string interactions, and the system can be described using semi-classical gravity.

Requiring translation symmetry on the brane leads to the following ansatz:

$$ds^2 = H(r)^{-1/2}\eta_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu + H(r)^{1/2}\delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j,$$

where μ, ν are brane direction and i, j are direction perpendicular to the brane . $r^2 = \sum_{i=4}^{9} x_i^2$ represents the distance to the brane.

Substituting this into the equations of motion of type IIB SUGRA, we obtain the solution

$$H(r) = 1 + \left(\frac{L}{r}\right)^4.$$

The parameter L is determined by$L^4 = 4\pi g_s\, N\alpha'^2$.

When $r \gg L$, we have the spacetime structure far from the branes, which is Minkowski space.

When $r \ll L$, we obtain the near-horizon geometry, also known as the throat:

$$\mathrm{d}s^2 = \frac{r^2}{L^2}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \frac{L^2}{r^2}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j.$$

Letting $z = \frac{L^2}{r}$, we have

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}z^2\right) + L^2\,\mathrm{d}s_s^2$$

This is the spacetime structure of $AdS_5 \times S^5$.

When discussing the energy of a gravitational system, we need to specify an observer. The energy E corresponds to ∂_t, where t is the proper time of an observer at infinity. Using a redshift factor, we describe the observer's energy as $E = H^{-1/4}E_p$. For a finite-sized system at r, in the limit $r \to 0$, E remains in the low-energy regime and $E \to 0$. Thus, under a given energy scale, the physics naturally **decouples** at $r \to 0$ and $r \to \infty$.

Summary

We understand the same system using two different approaches and obtain two **decoupled** systems in the limit of $\alpha' \to 0$. From the first perspective, we get free gravity in the bulk and $\mathcal{N} = 4$ gauge fields on the D_3-brane. From the second perspective, we get free gravity at large distances and a gravitational theory at $r \to 0$ close to the brane. Therefore, we consider $\mathcal{N} = 4$ SYM and AdS spacetime gravity to be two descriptions of the same physical system.

- Geometry Interpretation:

Our previous description of the gauge theory was in the context of flat spacetime. Now, in the limit $r \to 0$, we obtain the spacetime structure of $AdS_5 \times S^5$, and only by moving away from the throat do we return to a flat spacetime background. Therefore, we consider the gravity theory to live in $AdS_5 \times S^5$, while the Super Yang-Mills theory lives on the 4-dimensional Minkowski space, which is the boundary of $AdS_5 \times S^5$.

- Limits of Validity

When discussing supergravity, we need the curvature of the background spacetime to be much larger than the string scale, which means

$$R = \sqrt{\alpha'}(g_s N)^{1/4} \gg \sqrt{\alpha'} = l_s$$

resulting in $g_s N \gg 1$. The requirement for classical gravity solutions is that quantum corrections are small, corresponding to $g_s \to 0$.Therefore, we require $g_{s\to 0, N\to\infty}$ with $g_s N$ being large but finite.

References

[1] Johanna Erdmenger and Martin Ammon. Gauge/gravity duality, 2015.

[2] Juan Maldacena. The large-n limit of superconformal field theories and supergravity. *International journal of theoretical physics*, 38(4):1113–1133, 1999.

[3] Brian R Greene, David R Morrison, and Joseph Polchinski. String theory. *Proceedings of the National Academy of Sciences*, 95(19):11039–11040, 1998.

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